

**II B. Tech I Semester Supplementary Examinations, May/June - 2016**  
**COMPLEX VARIABLES AND STATISTICAL METHODS**  
 (Electrical and Electronics Engineering)

Time: 3 hours

Max. Marks: 70

- Note: 1. Question Paper consists of two parts (**Part-A** and **Part-B**)  
 2. Answer **ALL** the question in **Part-A**  
 3. Answer any **THREE** Questions from **Part-B**
- ~~~~~

**PART -A**

1. a) Show that the complex variable function  $f(z) = |z|^2$  is differentiable only at the origin.
  - b) Expand the following function in a Taylor's series  $\frac{z-1}{z+1}$  about  $z = 0$ .
  - c) Determine the poles and residues at each pole of the function  $f(z) = \cot z$
  - d) Find the image of following region in the  $z$ -plane onto the  $w$ -plane under the given mapping  $|z| < 1, \text{Im } z > 0; w = z + (2 + i)$
  - e) For the discrete probability distribution, find the value of  $K$
- |        |   |     |      |      |      |       |        |            |
|--------|---|-----|------|------|------|-------|--------|------------|
| $x$    | 0 | 1   | 2    | 3    | 4    | 5     | 6      | 7          |
| $f(x)$ | 0 | $K$ | $2K$ | $2K$ | $3K$ | $K^2$ | $2K^2$ | $7K^2 + K$ |
- f) An automobile manufacturer asserts that the seat belts of his seats are 90% effective. A consumer group tests the seat belts on 50 cars and finds it effective on 37 of them. What is the test statistic to be used to test his hypothesis?

**PART -B**

2. a) (i) Show that the function  $f(z) = \bar{z}$  is continuous everywhere but not differentiable at any point in the complex plane.  
 (ii) Show that  $f(z) = \bar{z}$  is not differentiable at  $z = 0$  and is nowhere analytic.
- b) Find the analytic function  $f(z) = u + iv$  given

$$u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$$

3. a) State and prove Cauchy's integral formula and hence find the value of  
 (i)  $F(3.5)$  (ii)  $F(i)$ , if

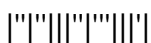
$$F(a) = \int_c \frac{4z^2 + z + 5}{z - a} dz \text{ where } c \text{ is the ellipse } \left(\frac{x}{2}\right)^2 + \left(\frac{y}{3}\right)^2 = 1.$$

- b) Expand  $f(z) = \frac{1}{z(z^2 - 3z + 2)}$  in the region  $0 < |z| < 1$

4. a) State residue theorem and use it to evaluate

$$\int_c \frac{dz}{z^3(z+4)} \text{ where } c \text{ is the circle (i) } |z|=2 \text{ (ii) } |z+2|=3$$

- b) Evaluate  $\int_0^{2\pi} \frac{d\theta}{a + b \cos \theta}$ ;  $a > |b| > 0$  and using it prove that  $\int_0^\pi \frac{d\theta}{17 - 8 \cos \theta} = \frac{\pi}{15}$ .



5. a) Find the image of the triangle with vertices at  $i$ ,  $1 + i$ ,  $1 - i$  in the  $z$ -plane under the transformation

$$w = e^{\frac{5\pi i}{3}} \cdot z - 2 + 4i$$

- b) Find the bilinear transformation which maps the points  $z = 0, 1, i$  in the  $z$ -plane onto the points  $1 + i, -i, 2 - i$  in the  $w$ -plane respectively.

6. a) Fit a normal distribution to the following data:

|           |       |       |       |       |       |
|-----------|-------|-------|-------|-------|-------|
| Class     | 60-62 | 63-65 | 66-68 | 69-71 | 72-74 |
| Frequency | 5     | 18    | 42    | 27    | 8     |

- b) A random sample of 100 mill workers at Kanpur showed their mean wage to be Rs. 3500 with a standard deviation of Rs. 280. Another random sample of 150 mill workers in Mumbai showed the mean wage to be Rs. 3900 with a standard deviation of Rs. 400. Do the mean wage of the workers in Mumbai and Kanpur differ significantly, at 5% level of significance?

7. a) Certain pesticides is packed into bags by a machine. A random sample of 10 bags is drawn and their contents are found to weigh in kgs as follows:  
50, 49, 44, 52, 45, 48, 46, 45, 49, 45.

Test if the average picking can be taken as 50 kg.

- b) The following data presents the yields in quintals of common 10 subdivisions of equal area of two agricultural plots:

|        |     |     |     |     |     |     |     |     |     |     |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| Plot1  | 6.2 | 5.7 | 6.5 | 6.0 | 6.3 | 5.8 | 5.7 | 6.0 | 6.0 | 5.8 |
| Plot 2 | 5.6 | 5.9 | 5.6 | 5.7 | 5.8 | 5.7 | 6.0 | 5.5 | 5.7 | 5.5 |

Test whether the two samples taken from the two random populations have the same variance.

