

**MATHEMATICS-III**

(Common to All Branches)

**Time: 3 hours**

**Max. Marks: 70**

Question Paper Consists of **Part-A** and **Part-B**  
 Answering the question in **Part-A** is Compulsory,  
 Three Questions should be answered from **Part-B**

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**PART-A**

1. (a) Find the Rank of the matrix  $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$  using Echelon form
- (b) Prove that the matrix A and  $A^T$  have same Eigen values
- (c) Find the volume of loop of the curve  $2ay^2 = x(x - a)^2$  revolves about x-axis
- (d) Evaluate  $\int_0^1 x^5 (1 - x^3)^{10} dx$
- (e) Prove that  $\text{div}(r \times a) = 0$  where a is a constant vector
- (f) Evaluate  $\int f \cdot dr$  where  $f = (2y + 3)i + xzj + (yz - x)k$  along the straight line joining (0,0,0) and (2,1,1)

[3+3+4+4+4+4]

**PART-B**

2. (a) Test for consistency and solve  $5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$ .
- (b) Solve the equations  
 $xy + z - w = 2, 7x + y + 3z + w = 12, 8x - y + z - 3w = 5, 10x + 5y + 3z + 2w = 20$ . by Gauss-Jordan method

[8+8]

3. (a) Verify Cayley Hamilton theorem for  $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ , hence compute  $A^4$  and  $A^{-1}$

- (b) Reduce the quadratic form  $3x^2 + 5y^2 + 3z^2 - 2yz + 2zx - 2xy$  in to canonical form by orthogonal reduction hence find rank, index and signature .

[8+8]

4. (a) Trace the curve  $x = a \cos^3 \theta, y = b \sin^3 \theta$

- (b) Evaluate the  $\int_0^a \int_{x/a}^{\sqrt{x/a}} (x^2 + y^2) dx dy$  by change of order of integration

[8+8]



5. (a) Prove that  $\nabla \cdot (\bar{f} \times \bar{g}) = \bar{g} \cdot (\nabla \times \bar{f}) - \bar{f} \cdot (\nabla \times \bar{g})$   
(b) Find the angle between the surfaces  $x^2 + y^2 - z^2 = 12$  &  $x^2 + y^2 - z = 5$  at  $(2, 2, 1)$  [8+8]

6. (a) Evaluate  $\iint_s x^3 dydz + x^2 y dzdx + x^2 z dxdy$  over the surface bounded by the planes  $z = 0, z = b$  and the cylinder  $x^2 + y^2 = a^2$ .  
(b) Evaluate  $\iiint_v 45x^2 y dx dy dz$  and  $v$  is the region bounded by  $x = y = z = 0$  and  $4x + 2y + z = 8$  [8+8]

7. (a) Evaluate  $\int_0^\infty 3^{-4x^2} dx$   
(b) Prove that  $\Gamma(n)\Gamma(1-n) = \pi / \sin n\pi$  [8+8]

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