

Subject Code: R13202/R13

Set No - 1

I B. Tech II Semester Regular/Supply Examinations July - 2015

**MATHEMATICS-III**

(Common to All Branches)

Time: 3 hours

Max. Marks: 70

Question Paper Consists of **Part-A** and **Part-B**  
Answering the question in **Part-A** is Compulsory,  
Three Questions should be answered from **Part-B**

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**PART-A**

- 1.(a) Find for what values of 'a' the equations,  $x + y + z = 1$ ,  $x + 2y + 4z = a$  and  $x + 4y + 10z = a^2$  have a solution.
- (b) Find the moment of inertia about the initial line of the cardioids  $r = a(1 + \cos\theta)$ .
- (c) What is the nature of the quadratic form  $X^TAX$ , if  $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ .
- (d) Evaluate  $\int_0^1 \frac{x dx}{\sqrt{1-x^5}}$ .
- (e) If  $\phi$  satisfies Laplace equation, show that  $\nabla\phi$  is both solenoidal and irrotational.
- (f) Use Greens theorem to evaluate  $\int_c (2xy - x^2)dx + (x^2 + y^2)dy$ , where  $c$  is the closed curve of the region bounded by  $y = x^2$  and  $y^2 = x$ .

[3+4+4+4+3+4]

**PART-B**

- 2.(a) Solve the system of equations  $8x - 3y + 2z = 20$ ,  $4x + 11y - z = 33$  and  $6x + 3y + 12z = 36$  using Gauss-Seidel method.
- (b) Reduce the matrix A to normal form and hence find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

[8+8]

- 3.(a) Find Eigen values and Eigen vectors of  $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ .

- (b) Reduce the Quadratic form to canonical form by orthogonal reduction and state the nature of the quadratic form  $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ .

[8+8]



4.(a) Find surface area of the right circular cone generated by the revolution of right angled triangle about a side which contains a right angle.

(b) Evaluate  $\int_0^a \int_{a-x}^{\sqrt{a^2-x^2}} y dx dy$  by changing the order of integration.

[8+8]

5.(a) Evaluate  $\int_0^{\infty} x^2 e^{-x^2} dx$  .

(b) Express  $\int_0^{\infty} \frac{x^c}{c^x} dx$ , ( $c > 1$ ) in terms of Gamma function.

[8+8]

6.(a) Find the directional derivative of the function  $\phi = xy^2 + yz^3$  at  $(2,-1,1)$  in the direction of normal to the surface  $x \log z - y^2 + 4 = 0$  at  $(-1,2,1)$ .

(b) Show that  $\frac{\vec{r}}{r^3}$  is solenoidal, where  $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ .

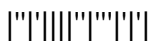
[8+8]

7.(a) If  $\vec{F} = xy\vec{i} - z\vec{j} + x^2\vec{k}$  and C is the curve  $x = t^2$ ,  $y = 2t$ , and  $z = t^3$  from  $t=0$  to  $t=1$ , find the work done by  $\vec{F}$  .

(b) Use divergence theorem to evaluate  $\iiint_S \vec{F} \cdot d\vec{S}$  where  $\vec{F} = 4x\vec{i} - 2y^2\vec{j} + z^2\vec{k}$  and S is the surface bounded by the region  $x^2 + y^2 = 4$ ,  $z=0$  and  $z=3$ .

[8+8]

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**PART-A**

- 1.(a) Find the value of 'a' for which the system of equations  $3x-y+ 4z =3$ ;  $x+2y -3z =-2$  and  $6x+5y +az = - 3$  will have infinite number of solutions.
- (b) If 2, 3, 5 are the eigenvalues of matrix A, then find the eigenvalues of  $2A^3+3 A^2 +5A +3I$ .
- (c) Find the moment of inertia about the initial line of the cardioid  $r = a(1- \cos\theta)$ .
- (d) Evaluate  $\int_0^1 \frac{x^3}{\sqrt{1-x^5}} dx$  in terms of Beta functions.
- (e) Find the angle between the surfaces  $x^2 + y^2 + z^2 = 9$  and  $z = x^2 + y^2 - 3$  at the point (2,-1, 2).
- (f) Evaluate  $\int_c (e^x dx + 2ydy - dz)$  where c is the curve is the curve  $x^2 + y^2 = 9, z=2$ , by using Stoke's theorem.

[3+3+4+4+4+4]

**PART-B**

- 2.(a) Solve the equations  $3x + y + 2z = 3, 2x - 3y - z = -3$  and  $x + 2y + z = 4$  using Gauss elimination method.
- (b) Reduce the matrix A to normal form and hence find the rank of the matrix

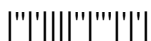
$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}$$

[8+8]

- 3.(a) Find  $A^{-1}$  using Cayley-Hamilton theorem , where  $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$ .

- (b) Reduce the Quadratic form  $x^2 + 3y^2 + 3z^2 - 2yz$  into canonical form and find the nature, rank, index and signature.

[8+8]



4.(a) Find the surface area generated by the revolution of an arc of the catenary  $y = c \cosh \frac{x}{c}$  about  $x$ -axis.

(b) Change the order of integration and evaluate  $\int_0^a \int_x^a (x^2 + y^2) dy dx$ .

[8+8]

5.(a) Prove that  $\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi}$

(b) Express  $\int_0^1 x^m (1-x^n)^p$  in terms of  $\Gamma$  function.

[8+8]

6.(a) Find the directional derivative of  $\frac{1}{r}$  in the direction of  $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$  at (1,1,2).

(b) If  $\bar{A}$  is irrotational, evaluate  $\text{div}(\bar{A} \times \bar{r})$  where  $\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ .

[8+8]

7.(a) Find the work done by the force  $z\bar{i} + x\bar{j} + y\bar{k}$ , if it moves a particle along the arc of the curve  $\bar{r} = \cos t\bar{i} + \sin t\bar{j} - t\bar{k}$  from  $t = 0$  to  $2\pi$ .

(b) Compute  $\int (ax^2 + by^2 + cz^2) ds$  over the surface of the sphere  $x^2 + y^2 + z^2 = 1$ .

[8+8]

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**PART-A**

1.(a) Find for what values of 'a' such that the rank of the matrix A is 2, where

$$A = \begin{bmatrix} 1 & 1 & -1 & 1 \\ 1 & -1 & a & -1 \\ 3 & 1 & 0 & 1 \end{bmatrix}$$

(b) Prove that the Eigen values of a Skew-Hermitian matrix are either purely imaginary or zero.

(c) Find the length of the curve  $3x^2 = y^3$  between  $y=0$  and  $y=1$ .

(d) Evaluate  $\int_0^1 \frac{dx}{(1-x^3)^{1/3}}$  using Beta functions.

(e) Find  $\text{div } \vec{F}$ , where  $\vec{F} = r^n \vec{r}$ . Find  $n$  if it is solenoidal.

(f) Using Stoke's theorem, evaluate the integral  $\int_c \vec{F} \cdot d\vec{r}$ , where

$\vec{F} = 2y^2\vec{i} + 3x^2\vec{j} - (2x+z)\vec{k}$  and  $c$  is the boundary of the triangle whose vertices are  $(0,0,0)$ ,  $(2,0,0)$  and  $(2,2,0)$ .

[3+3+4+4+4+4]

**PART- B**

2.(a) Using Gauss-Jordon method solve the system of equations

$$2x + y + z = 10, 3x + 2y + 3z = 18 \text{ and } x + 4y + 9z = 16 .$$

(b) Reduce the matrix A to normal form and hence find the rank of the matrix

$$A = \begin{bmatrix} 2 & 1 & 3 & 4 \\ 0 & 3 & 4 & 1 \\ 2 & 3 & 7 & 5 \\ 2 & 5 & 11 & 6 \end{bmatrix}$$

[8+8]



3.(a) Find  $A^{-1}$  by using Cayley-Hamilton theorem, where  $A = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 1 & -2 \\ 2 & -2 & 1 \end{bmatrix}$ .

(b) Reduce the Quadratic form to canonical form by orthogonal reduction and state the nature of the quadratic form  $2x^2 + 2y^2 + 2z^2 - 2xy - 2yz - 2zx$ . [8+8]

4.(a) Find the volume obtained by revolving the loop of the curve  $x = t^3, y = t - \frac{t^3}{3}$  about x-axis.

(b) Change the order of integration and evaluate  $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2 dy dx$ . [8+8]

5.(a) Evaluate  $\int_0^1 \frac{x^2}{\sqrt{1-x^5}} dx$  using  $\beta$  and  $\Gamma$  function.

(b) Show that  $\int_0^\infty x^m e^{-ax^n} dx = \frac{1}{na^{\frac{m+1}{n}}} \Gamma\left(\frac{m+1}{n}\right)$ . [8+8]

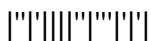
6.(a) Find the directional derivative of  $x^2 - 2y^2 + 4z^2$  at  $(1,1,-1)$  in the direction of  $2\bar{i} + \bar{j} - \bar{k}$

(b) Find  $a, b, c$  so that  $\bar{A} = (x + 2y + az)\bar{i} + (bx - 3y - z)\bar{j} + (4x + y + 2z)\bar{k}$  is irrotational. Also find  $\phi$  such that  $\bar{A} = \nabla\phi$ . [8+8]

7.(a) Compute the line integral  $\int (y^2 dx - x^2 dy)$  round the triangle whose vertices are  $(1,0), (0,1)$  and  $(-1,0)$ .

(b) Use divergence theorem to evaluate  $\iiint_S \bar{F} \cdot d\bar{S}$  where  $\bar{F} = x^3\bar{i} + y^3\bar{j} + z^3\bar{k}$  and S is surface of the sphere  $x^2 + y^2 + z^2 = r^2$ . [8+8]

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**PART-A**

- 1.(a) Find the values of 'a' and 'b' for which equation  $x + y + z = 3; x + 2y + 2z = 6; x + ay + 3z = b$  have unique solution.
- (b) Prove that the eigenvalues of a Skew-Hermitian matrix are either purely imaginary or zero.
- (c) Find the moment of inertia about the initial line of the cardioid  $r = a(1 + \cos\theta)$ .
- (d) Evaluate  $\int_0^1 \frac{x^3}{\sqrt{1-x^5}} dx$  in terms of Beta functions.
- (e) Find the directional derivative of  $2xy + z^2$  at  $(1, -1, 3)$  in the direction of  $\bar{i} + 2\bar{j} + 3\bar{k}$
- (f) Using Stoke's theorem, evaluate  $\int_c \bar{F} \cdot d\bar{r}$ , where  $\bar{F} = 2y^2\bar{i} + 3x^2\bar{j} - (2x + z)\bar{k}$  and  $c$  is the boundary of the triangle whose vertices are  $(0,0,0), (1,0,0)$  and  $(1,1,0)$ .

[3+3+4+4+4+4]

**PART-B**

- 2.(a) Solve the system of equations using Gauss-Seidel method correct to three decimal place  $8x - 3y + 2z = 20, 4x + 11y - z = 33$  and  $6x + 3y + 12z = 36$ .
- (b) Reduce the matrix A to normal form and hence find the rank of the matrix

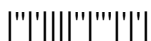
$$A = \begin{bmatrix} 2 & 1 & 3 & 5 \\ 4 & 2 & 1 & 3 \\ 8 & 4 & 7 & 13 \\ 8 & 4 & -3 & -1 \end{bmatrix}$$

[8+8]

- 3.(a) Find Eigen values and Eigen vectors of  $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ .

- (b) Reduce the Quadratic form  $10x^2 + 2y^2 + 5z^2 - 4xy - 10xz + 6yz$  into canonical form and find the nature, rank, index and signature.

[8+8]



4.(a) Find the volume of the solid of revolution generated by the revolution of the cissoid

$$y^2 = \frac{x^3}{2a-x} \text{ about its asymptote.}$$

(b) Change the order of integration and evaluate  $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy^2 dy dx$ .

[8+8]

5.(a) Evaluate  $\int_5^7 (x-5)^6 (7-x)^3 dx$  using  $\beta$  and  $\Gamma$  functions.

(b) Express  $\int_0^1 \frac{dx}{\sqrt{1-x^n}}$  in terms of  $\Gamma$  function.

[8+8]

6.(a) If Prove that  $\nabla \cdot \left[ r \nabla \left( \frac{1}{r^3} \right) \right] = \frac{3}{r^4}$ , where  $r = \sqrt{x^2 + y^2 + z^2}$ .

(b) Find the directional derivative of  $x^2 - 2y^2 + 4z^2$  at (1, 1,-1) in the direction of  $2\bar{i} + \bar{j} - \bar{k}$ .

[8+8]

7.(a) If  $\bar{F} = (5xy - 6x^2)\bar{i} + (2y - 4z)\bar{j}$ , evaluate  $\int_c \bar{F} \cdot d\bar{r}$  along the curve  $c: y=x^3$  from (1,1) to (2,8).

(b) Apply Stoke's theorem to evaluate  $\oint_c (ydx + zdy + xdz)$  where  $c$  is the curve of intersection of the sphere  $x^2 + y^2 + z^2 = a^2$  and  $x+z = a$ .

[8+8]

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